MACM 401/MATH 701/MATH 819 Assignment 6, Spring 2009.

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This assignment is to be handed in by Thursday April 9th.
For problems involving Maple calculations and Maple programming, you should submit a printout of a Maple worksheet of your Maple session.
Late Penalty: -20% for up to 24 hours late. Zero after that.

Question 1: Polynomial Resultants (20 marks)

Consider the following pairs of polynomials

$$f_1 = 2x^3 + 2x - x^2 - 1,\ g_1 = x^2 - 2$$

$$f_2 = (x^2 + 1 - x)z + (x^4 + 2\,x^2 - x),\ g_2 = z^2 + 1 \ \text{ and}$$

$$f_3 = 2yx^3 - x^2y^2 + (2y + 1)x - 3,\ g_3 = 2x^2 - (3 - y^2)x + (2y^2 - 5).$$

Compute the resultants $\texttt{res}_x(f_1, g_1)$, $\texttt{res}_z(f_2, g_2)$ and $\texttt{res}_x(f_3, g_3)$ (please note which variable is being eliminated!) using the Maple command `resultant` (so that you know what the answers are). Now modify the natural Euclidean algorithm to compute the resultant. Program your algorithm in Maple. Execute it on the above inputs. Now modify the primitive Euclidean algorithm to compute the resultant. Program your algorithm in Maple. Execute it on the above inputs.

Question 2: Rational Function Integration (20 marks)

Reference: sections 11.3, & 11.4 and 11.5.

(a) Calculate the Trager-Rothstein resultant for the rational function integral below by hand by constructing Sylvester's matrix for the resultant and computing the determinant by hand. Complete the integral.

$$\int \frac{2x+1}{x^2-2}\mathrm{dx}.$$

(b) Integrate the three rational functions below as follows. First compute the rational function part of integral using either Hermite's method or Horrowitz's method (or both if you wish). Then compute the logarithmic part (if any) using the Trager-Rothstein resultant. Use Maple to help with arithmetic e.g. you may use `gcd`, `gcdex`, `solve`, `resultant` etc.

$$f_1 = \frac{3\,x^5 - 2\,x^4 - x^3 + 2\,x^2 - 2\,x + 2}{x^6 - x^5 + x^4 - x^3}$$

$$f_2 = \frac{4\,x^7 - 16\,x^6 + 28\,x^5 - 351\,x^3 + 588\,x^2 - 738}{2\,x^7 - 8\,x^6 + 14\,x^5 - 40\,x^4 + 82\,x^3 - 76\,x^2 + 120\,x - 144}$$

$$f_3 = \frac{6\,x^5 - 4\,x^4 - 32\,x^3 + 12\,x^2 + 34\,x - 24}{x^6 - 8\,x^4 + 17\,x^2 - 8}$$

Reading Assignment (0 marks)

Study examples 12.2, 12.3, 12.4 and 12.5 and the proofs of Theorem 12.3 and Theorem 12.4 (Liouville's theorem – the special cases only).

Question 3: Non-elementary Integrals (20 marks)

Reference: 12.6, 12.7

Apply the Risch algorithm to prove that the following integrals are not elementary.

$$\int \frac{e^x}{x} \mathrm{d}x$$

$$\int \frac{1}{\log(x)^2} \mathrm{d}x$$

$$\int \frac{\log(x)}{x+1} \mathrm{d}x$$

$$\int e^x \log(x) \mathrm{d}x$$

Question 4: Elementary Integrals (20 marks)

Reference: 12.6, 12.7

Apply the Risch algorithm to compute the following elementary integrals.

$$\int \frac{e^{2x}}{e^{2x} + e^x + 1} \, \mathrm{d}x$$

$$\int 2\theta + 2 - \frac{1/x + 1}{(\theta + x)^2} + \frac{1}{x\theta} \, \mathrm{d}x \qquad \text{where} \quad \theta = \log(x)$$

$$\int \theta + x\theta + \frac{2}{x}\theta^2 - \frac{1}{x^2}\theta^2 \, \mathrm{d}x \qquad \text{where} \quad \theta = e^x$$