MATH 340 Assignment 6, Fall 2008

Michael Monagan

This assignment is due Friday November 7th at 11:00am.
Late penalty: -20% for handing in by 11:00am, Monday November 10th. Zero after that.
For problems involving Maple please submit a printout of a Maple worksheet.
There will be a Maple tutorial on Tuesday November 4th @ 8:30am and 9:30am.

Primitive Roots of Unity

1. Sketch the 12'th roots of unity in the complex plane. Circle which ones are primitive in your sketch and determine formulae for the primitive 12'th roots of unity.

2. Let ω be a primitive n'th root of unity.
   For n even, prove that $\omega^{n/2} = -1$.

3. Let ω be a primitive n'th root of unity. In class we proved for n even, $\sum_{i=1}^{n} \omega^i = 0$ and $\Pi_{i=1}^{n} \omega^i = -1$. For n odd, using Maple to assist you, determine the values of $\sum_{i=1}^{n} \omega^i$ and $\Pi_{i=1}^{n} \omega^i$. Now prove your results.

Section 2.7: Construction of Finite Fields

Exercises 1, 5, 6, 7, 8, 9.

Section 2.8: Extension Fields

Exercises 2, 3, 4, 5, 9.
For question 5 – show that $x^3 - x + 1$ has no roots in GF(9).
Do question 9 – construct GF(8) – in Maple. Explicitly construct the addition and multiplication tables.

Additional questions on extension fields.

1. Is $\mathbb{Q}[z]/(z^3 + 1)$ a field? Justify your answer briefly.

2. Consider the field $F = \mathbb{Q}[z]/(z^2 - 2)$. What is the inverse of $[z] \in F$?

3. Consider the field $F = \mathbb{R}[z]/(z^2 + 1)$. This field is isomorphic to another field K that we have already seen. What is K and what is the isomorphism $\psi : F \to K$?