MACM 401/MATH 701/MATH 819 Assignment 6, Spring 2007.

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This assignment is to be handed in by Thursday April 12th. For problems involving Maple calculations and Maple programming, you should submit a printout of a Maple worksheet of your Maple session. Late Penalty: -20% for each day late.

Question 1: Polynomial Resultants (10 marks)

Consider the following pairs of polynomials

$$f_1 = 2x^3 + 2x - x^2 - 1, \; g_1 = x^2 - 2$$

$$f_2 = (x^2 + 1 - x)z + (x^4 + 2x^2 - x), \; g_2 = z^2 + 1 \;\text{ and}$$

$$f_3 = 2yx^3 - x^2y^2 + (2y+1)x - 3, \; g_3 = 2x^2 - (3 - y^2)x + (2y^2 - 5).$$

Compute the resultants $\texttt{res}_x(f_1, g_1)$, $\texttt{res}_z(f_2, g_2)$ and $\texttt{res}_x(f_3, g_3)$ (please note which variable is being eliminated!) using the Maple command $\texttt{resultant}$ (so that you know what the answers are). Now modify the Euclidean algorithm to compute the resultant. Program your algorithm in Maple. Execute it on the above inputs.

Question 2: Rational Function Integration (30 marks)

Reference: sections 11.3, & 11.4 and 11.5.

To integrate the three functions below, first compute the rational function part of the following integrals using either Hermite's method or Horrowitz's method (or both if you wish). Then compute the logarithmic part using the Trager-Rothstein resultant. Use Maple to help with arithmetic e.g. you may use $\texttt{gcd}$, $\texttt{gcdex}$, $\texttt{solve}$, $\texttt{resultant}$ etc.

$$f_1 = \frac{3x^5 - 2x^4 - x^3 + 2x^2 - 2x + 2}{x^6 - x^5 + x^4 - x^3}$$

$$f_2 = \frac{4x^7 - 16x^6 + 28x^5 - 351x^3 + 588x^2 - 738}{2x^7 - 8x^6 + 14x^5 - 40x^4 + 82x^3 - 76x^2 + 120x - 144}$$

$$f_3 = \frac{6x^5 - 4x^4 - 32x^3 + 12x^2 + 34x - 24}{x^6 - 8x^4 + 17x^2 - 8}$$

Now consider $\int \frac{2x+1}{x^2-2}dx$. Calculate the Trager-Rothstein resultant for this integral by hand by constructing sylvester's matrix for the resultant and computing the determinant by hand.

Question 3: Non-elementary Integrals (20 marks)

Reference: 12.6, 12.7

    Apply the Risch algorithm to prove that the following integrals are not elementary.

$$\int \frac{e^x}{x}\,\mathrm{d}x$$

$$\int \frac{1}{\log(x)^2}\,\mathrm{d}x$$

$$\int \frac{\log(x)}{x+1}\,\mathrm{d}x$$

$$\int e^x \log(x)\,\mathrm{d}x$$

Question 4: Elementary Integrals (20 marks)

Reference: 12.6, 12.7

    Apply the Risch algorithm to compute the following elementary integrals.

$$\int \frac{e^{2x}}{e^{2x} + e^x + 1}\,\mathrm{d}x$$

$$\int 2\theta + 2 - \frac{1/x+1}{(\theta+x)^2} + \frac{1}{x\theta}\,\mathrm{d}x \qquad \text{where} \ \ \theta = \log(x)$$

$$\int \theta + x\theta + \frac{2}{x}\theta^2 - \frac{1}{x^2}\theta^2\,\mathrm{d}x \qquad \text{where} \ \ \theta = e^x$$