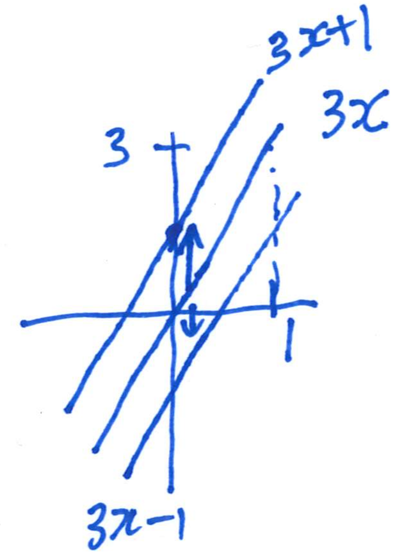
MATH 152.

4.9 Antiderivatives.

$g(x) = f'(x)$	$f(x) \leftarrow$ an antiderivative
3	$3x$, $3x+1$, $3x+C$
$2x$	x^2+C
$2+x$	$2x+\frac{1}{2}x^2+C$
$\cos x+1/x$	$\sin x+\ln x+C$
$2\cos \overset{u}{2x}$	$\sin(\overset{u}{2x})+C$



These all have the same slope.

Theorem. If $f'(x)=g'(x)$. then $f(x)=g(x)+C$ for some constant C.

Definition. If $f'(x)=g(x)$ then $f(x)$ is an <u>antiderivative</u> of $g(x)$.
and $f(x)+C$ is the <u>general antiderivative</u> of $g(x)$.

Ex 1. Find the general antiderivative of $2x+1$.

$$f(x)=x^2+x+C$$

What's C? We can determine C if we know any value of $f(x)$ e.g. $f(0)=2$.

Ex 2. Given $f'(x) = 6x$ and $\boxed{f(0)=2}$ find $f(x)$.

$f(x) = 3x^2 + C$

$f(0) = \boxed{0 + C = 2} \Rightarrow C = 2$

$\Rightarrow f(x) = 3x^2 + 2$.

Ex. 3 Given $f''(x) = 6x$ and $\boxed{f(0)=3}$ and $\boxed{f'(0)=0}$ find $f(x)$.

$f'(x) = 3x^2 + C$

$f'(0) = C = 0 \Rightarrow C = 0$

$f'(x) = 3x^2$

$f(x) = x^3 + b$ for some constant b.

$f(0) = 0 + b = 3 \Rightarrow b = 3$.

$f(x) = x^3 + 3$.

Ex 4. Given $f''(x) = \cos x$, $f(0) = 0$, $f'(0) = 0$ find $f(x)$.

Table of Antiderivatives

function	antiderivative		function	antiderivative.
e^x	e^x			
$\cos x$	$\sin x$			
$\sin x$	$-\cos x$			
x^n	$\frac{1}{n+1} x^{n+1}$	$n \neq -1$	x^2	$\frac{1}{3} \cdot x^3$
$x^{-1} = \frac{1}{x}$	$\ln x$		$\sqrt{x} = x^{\frac{1}{2}}$	$\frac{1}{3/2} \cdot x^{3/2} = \boxed{\frac{2}{3} x^{3/2}}$
a	$a \cdot x$			
$a \cdot \boxed{f'(x)}$	$a \cdot f(x)$		$2 \cdot x^1$	$2 \cdot \left(\frac{1}{2} x^2\right) = x^2$
$f'(x) + g'(x)$	$\xleftarrow{\prime} f(x) + g(x)$		$e^x + x$	$e^x + \frac{1}{2} x^2$
$\sec^2 x$	$\longrightarrow \tan x$			

Linear Motion Let $d(t)$, $v(t)$, $a(t)$ be the distance travelled, velocity and acceleration of an object at time t. Then

$$\boxed{v(t) = d'(t) \quad \text{and} \quad a(t) = v'(t).}$$

Ex. If a stone is dropped from a height of 490m above the ground, how long does it take to hit the ground?
How fast is it going when it hits the ground?

Let $d(t)$ be the height of the stone above the ground at time t.

490m

0m. $d(t)=0$.

$\boxed{d(0) = 490m}$

$\boxed{v(0) = 0}$

$a(t) = -9.8 m/s^2$

$a(t) = \boxed{v'(t) = -9.8}$

$v(t) = -9.8t + C$

$v(0) = 0 + C = 0$

$\Rightarrow v(t) = -9.8t$

$v(t) = d'(t) = -9.8t$

$d(t) = -9.8\left(\frac{1}{2}t^2\right) = -4.9t^2 + b$

$d(0) = \quad = 0 + b = 490 m$

$\Rightarrow d(t) = -4.9t^2 + 490.$

$d(t) = 0 = 490 - 4.9t^2$

$\Rightarrow 490 = 4.9t^2$

$\Rightarrow t = 10s$

$v(10) = -9.8 \cdot 10 = -98 m/s.$

Sigma Notation.

upper index

summand

Def The sum $\sum_{i=m}^{n} f(i) = f(m)+f(m+1)+f(m+2)+\cdots+f(n)$.

index → $i=m$ ← lower index

Ex. $\sum_{i=0}^{3}(2i+1) = (\underset{i=0}{1} + \underset{i=1}{3} + \underset{i=2}{5} + \underset{i=3}{7}) = 16.$

Properties ① $\sum_{i=m}^{n} c\,f(i) = c\sum_{i=m}^{n} f(i)$ ② $\sum_{i=m}^{n}(f(i)+g(i)) = \sum_{i=m}^{n} f(i) + \sum_{i=m}^{n} g(i)$

Ex $\sum_{i=0}^{3}(2i+1) \overset{②}{=} \sum_{i=0}^{3} 2i + \boxed{\sum_{i=0}^{3} 1} \overset{①}{=} 2\sum_{i=0}^{3} i + \sum_{i=0}^{3} 1 =$

$= 2(\boxed{0+1+2+3}) + (1+1+1+1)$

$= 12+4=16.$

Two useful formulas

F1 $\sum_{i=1}^{n} i = 1+2+3+\cdots+n = \frac{n(n+1)}{2}$

F2 $\sum_{i=1}^{n} i^2 = 1^2+2^2+3^2+\cdots+n^2 = \frac{n}{6}(n+1)(2n+1).$

$\sum_{i=0}^{3} i = 0+\underline{1+2+3} = \boxed{1+2+3} = \sum_{i=1}^{3=n} i = \frac{3(3+1)}{2} = \frac{3\cdot 4}{2} = 6$