MATH 340 Assignment 5, Fall 2008

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This assignment is due Wednesday October 29th at the beginning of class.
For problems involving Maple please submit a printout of a Maple worksheet.
Late penalty: -20% for up to 24 hours late. Zero for more than 24 hours late.

Section 2.3: Review of Vector Spaces

Exercises 2, 7, 9.
Also exercise 12 of section 1.7 and exercise 18 of section 2.1.

Section 2.6: Irreducible Polynomials

Exercises 1, 2, 6, 9, 10.
Do questions 1, 2 and 9 by hand. Use Maple to answer question 10.

Complex numbers

1. Let $i = \sqrt{-1}$, $a = (2+3i)$ and $b = (1-2i)$. Calculate $a+b$, $a\,b$ and a/b.
2. Convert $a = 2-2i$ and $b = 2i$ to polar co-ordinates and calculate a^2, ab and a/b in polar form. By hand, draw the points a, b, a^2, ab and a/b in the complex plane.
3. Let $z_1 = r(\cos\theta + i\sin\theta)$ and $z_2 = s(\cos\omega + i\sin\omega)$.
   Show that $z_1/z_2 = r/s\,[\cos(\theta-\omega) + i\sin(\theta-\omega)]$.
4. Let $\mathbb{Z}[i] = \{a+bi : a, b \in \mathbb{Z} \text{ and } i^2 = -1\}$ and let addition and multiplicaton in $\mathbb{Z}[i]$ be defined as for $\mathbb{C}$. The set $\mathbb{Z}[i]$ is called the set of Gaussian integers. Prove that $\mathbb{Z}[i]$ is a subring of $\mathbb{C}$. See Lemma 2.2.4.

The Fundamental Theorem of Algebra

1. By hand, calculate the real and complex eigenvalues AND eigenvectors of the matrix
$$\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$
2. Using Maple, factor the polynomials x^4-3x^2+1 and x^4+x^2+1 over $\mathbb{Q}$. Then determine all real and complex roots by applying the quadratic formula by hand.
3. Determine all real and complex roots of the polynomials x^4+3x^2+1 and x^5-1 in Maple using both the `solve` and `fsolve` commands.
   Observe that $f(a+bi) = 0 \implies f(a-bi) = 0$.