MATH 340 Assignment 5, Fall 2017

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This assignment is due Wednesday November 1st at 11:20am.
For problems involving Maple please submit a printout of a Maple worksheet.
Late penalty: -20% for up to 48 hours late. Zero for more than 48 hours late.

Section 2.5: Complex Numbers

1. Let $i^2 = -1$, $a = (2 + 3i)$ and $b = (1 - 2i)$.
   Calculate $a + b$, $a\,b$, a^{-1}, $|a|$ and $\bar{b}$.
   Draw the points $a, b, a\,b, a^{-1}, \bar{b}$ in the complex plane.

2. Let $x, y \in \mathbb{C}$. Show that $xy = yx$ and $|xy| = |x||y|$ and $\overline{xy} = \bar{x}\,\bar{y}$.

3. If $f(x) = x^3 - 6x^2 + 13x - 10$ and $2 + i$ is a root of $f(x)$, find the other roots of $f(x)$ and factor $f(x)$ over $\mathbb{C}$.

4. Let $\mathbb{Z}[i]$ be the subset of complex numbers $\mathbb{C}$ given by $\mathbb{Z}[i] = \{a + bi : a, b \in \mathbb{Z}$ and $i^2 = -1\}$. The set $\mathbb{Z}[i]$ is called the set of Gaussian integers. Show that $\mathbb{Z}[i]$ is a subring of $\mathbb{C}$. See Lemma 2.2.4 (i).

Section 2.6: Irreducible Polynomials

Exercises 1, 2, 9, 10, 13, 14.

Section 2.7: Construction of Fields

Exercises 1, 2, 5, 6, 7, 8, 9.

Consider the ring $R = \mathbb{Z}_2[x]/(x^3 + x^2 + x)$.

(i) What are the congruence classes of R?

(ii) Find a zero divisor in R.

(iii) Use the extended Euclidean algorithm to find $[x + 1]^{-1}$ in R.

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