MATH 340 Assignment 6, Fall 2017

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This assignment is due Friday November 10th at 11:20am.
Late penalty: -20% for handing in by 11:20am, Tuesday November 14th. Zero after that.

Section 2.8: Extension Fields

Exercises 1, 3, 4, 9, 13.

For exercises 1,3,4 use the construction $GF(9) = \mathbb{Z}_3[x]/(x^2+1)$ as shown in example 2.8.1. In exercise 3, note that the polynomial $x^9 - x$ factors as $x(x^4-1)(x^4+1)$ and x^2+x+2 is an irreducible factor of x^4+1 over $\mathbb{Z}_3$.

Additional questions on extension fields and roots of unity.

1. Is $\mathbb{Q}[z]/(z^3+1)$ a field? Justify your answer briefly.
2. Consider the field $F = \mathbb{Q}[z]/(z^2-2)$.
   Use the extended Euclidean algorithm to find the inverse of $[z] \in F$.
3. Consider the field $F = \mathbb{R}[z]/(z^2+1)$. Let $\phi : F \to \mathbb{C}$ be the mapping given by $\phi([a+bz]) = a+bi$. Show that ϕ is isomorphism.
4. Sketch the 12'th roots of unity in the complex plane. Circle which ones are primitive.
5. Let ω be a primitive n'th root of unity. For n even, prove that $\omega^{n/2} = -1$.
6. What are the primitive 6'th roots of unity?
   Find $\phi_6(x)$ the sixth cyclotomic polynomial. See Theorem 2.8.11.

Section 2.9: Multiplicative Structure of Finite Fields

Exercises 1(ii), 2, 5.

Section 2.10: Primitive Elements

Exercises 1, 4, 5.

Using your multiplication table for $GF(8)$ from exercise 2.8 #9, calculate the order of all elements and determine which elements are primitive.