```
> f := y^2+1;
```

$$f := y^2 + 1$$

```
> F := [0,1,2,y,y+1,y+2,2*y,2*y+1,2*y+2];
```

$$F := [0, 1, 2, y, y+1, y+2, 2y, 2y+1, 2y+2]$$

```
> g := z^2+z+2;
```

$$g := z^2 + z + 2$$

```
> G := [0,1,2,z,z+1,z+2,2*z,2*z+1,2*z+2];
```

$$G := [0, 1, 2, z, z+1, z+2, 2z, 2z+1, 2z+2]$$

To find an isomorphism $\varphi := F \to G$ we need to find a root of f in G

```
> for beta in G do
    if Rem( eval(f,y=beta), g, z ) mod 3 = 0 then print(beta) fi;
  od;
```

$$z+2$$

$$2z+1$$

```
> beta := 2*z+1;
```

$$\beta := 2z + 1$$

```
> phi := a -> Rem(eval(a,y=beta),g,z) mod 3:
> F;
```

$$[0, 1, 2, y, y+1, y+2, 2y, 2y+1, 2y+2]$$

```
> map(phi,F);
```

$$[0, 1, 2, 2z+1, 2z+2, 2z, z+2, z, z+1]$$

Here is the multiplication table for G

```
> MG := Matrix(9,9):
  for i to 9 do for j to 9 do
     MG[i,j] := Rem(G[i]*G[j],g,z) mod 3
  od od:
  MG;
```

$$\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 2 & z & z+1 & z+2 & 2z & 2z+1 & 2z+2 \\
0 & 2 & 1 & 2z & 2z+2 & 2z+1 & z & z+2 & z+1 \\
0 & z & 2z & 2z+1 & 1 & z+1 & z+2 & 2z+2 & 2 \\
0 & z+1 & 2z+2 & 1 & z+2 & 2z & 2 & z & 2z+1 \\
0 & z+2 & 2z+1 & z+1 & 2z & 2 & 2z+2 & 1 & z \\
0 & 2z & z & z+2 & 2 & 2z+2 & 2z+1 & z+1 & 1 \\
0 & 2z+1 & z+2 & 2z+2 & z & 1 & z+1 & 2 & 2z \\
0 & 2z+2 & z+1 & 2 & 2z+1 & z & 1 & 2z & z+2
\end{bmatrix}$$

Here is the multipilcation table for F = $Z_3[y]/(y^2+1)$

```
> MF := Matrix(9,9):
  for i to 9 do for j to 9 do
     MF[i,j] := Rem(F[i]*F[j],f,y) mod 3
  od od:
  MF;
```

$$\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 2 & y & y+1 & y+2 & 2y & 2y+1 & 2y+2 \\
0 & 2 & 1 & 2y & 2y+2 & 2y+1 & y & y+2 & y+1 \\
0 & y & 2y & 2 & y+2 & 2y+2 & 1 & y+1 & 2y+1 \\
0 & y+1 & 2y+2 & y+2 & 2y & 1 & 2y+1 & 2 & y \\
0 & y+2 & 2y+1 & 2y+2 & 1 & y & y+1 & 2y & 2 \\
0 & 2y & y & 1 & 2y+1 & y+1 & 2 & 2y+2 & y+2 \\
0 & 2y+1 & y+2 & y+1 & 2 & 2y & 2y+2 & y & 1 \\
0 & 2y+2 & y+1 & 2y+1 & y & 2 & y+2 & 1 & 2y
\end{bmatrix}$$

Here is the multiplication table for G = $\mathbb{Z}_3[z]/(z^2+z+2)$
permuted by $\varphi(y) = 2\cdot z+1$

```
> M2 := Matrix(9,9):
  for i to 9 do for j to 9 do
     M2[i,j] := Rem(phi(F[i])*phi(F[j]),g,z) mod 3
  od od:
  M2;
```

$$\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 2 & 2z+1 & 2z+2 & 2z & z+2 & z & z+1 \\
0 & 2 & 1 & z+2 & z+1 & z & 2z+1 & 2z & 2z+2 \\
0 & 2z+1 & z+2 & 2 & 2z & z+1 & 1 & 2z+2 & z \\
0 & 2z+2 & z+1 & 2z & z+2 & 1 & z & 2 & 2z+1 \\
0 & 2z & z & z+1 & 1 & 2z+1 & 2z+2 & z+2 & 2 \\
0 & z+2 & 2z+1 & 1 & z & 2z+2 & 2 & z+1 & 2z \\
0 & z & 2z & 2z+2 & 2 & z+2 & z+1 & 2z+1 & 1 \\
0 & z+1 & 2z+2 & z & 2z+1 & 2 & 2z & 1 & z+2
\end{bmatrix}$$