

Perturbations of roots of polynomials

Branko Ćurgus
Department of Mathematics
Western Washington University
Bellingham, Washington

Let \mathcal{P}_n be the vector space of all polynomials of degree $\leq n$. Let A be a linear transformation on \mathcal{P}_n . The following question will be discussed:

Can we identify a region in the complex plane which contains all the roots of the polynomial Ap , provided that all the roots of a polynomial $p \in \mathcal{P}_n$ are given?

The celebrated Gauss-Lucas theorem answers this question when $A(p) = p'$. A theorem of Cauchy addresses a special case when A is a linear combination of derivatives and $p(x) = x^n$. Improvements of each of these results, as well as new results identifying a class of transformations A for which the above question has a nice answer will be presented.