Advanced Computational Mathematics on the Web

Theme 3.30 — By J Borwein, J Chang, A Cooper, S Dugaro, L Jörgenson, and T Stanway (CECM, SFU)

www.cecm.sfu.ca/TLRN/

introduction

Our project has concentrated on linking sophisticated mathematical tools flexibly within web-browsers. Our approach has been to develop resources that educators can adapt to the needs of their students. We have been exploring ways to get these resources to talk to each other and how to make them easy to use in the classroom or home. We provide examples here to introduce interfaces that can be adapted for users with a wide range of backgrounds in mathematics. These examples also serve to illustrate how different interfaces may communicate with each other and how they may be used together. In an effort to see these technologies gain rapid acceptance, we are committed to an *open source* philosophy wherever possible.

Examples of Doing Advanced Computational Mathematics Using Web-based Resources

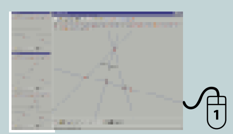
The Altitude of an Equilateral Triangle

1.0 Introduction

This activity uses a dynamic geometry package and the *Integer Relations Interface* to help students investigate the relationship between the altitude of an equilateral triangle and the length of one of its sides. This exercise is appropriate for middle school students because it ties together aspects of geometry with concepts of algebra that are needed in solving simple quadratic equations.

1.1 Using Cinderella

Using a dynamic geometry program such as *Cinderella*, students can draw an equilateral triangle, measure the length of its sides, and measure its altitude.



1.2 Using Integer Relations

From the measurements obtained in Cinderella, students may enter the ratio of the length of the altitude to the length of a side of the equilateral triangle in the *Evaluate* field.

1.2.1 Using Integer Relations as a Class Activity

Integer Relations evaluates the ratio by converting it to a decimal. The result is expressed in the *Alpha* field. Teachers should then encourage students to consider the effect of the *Degree* and *Precision* fields through experimenting with different values.

1.2.2 Using the Minimal Polynomial Panel

If the students enter a degree of 2 for the minimal polynomial and reasonable values for precision, Integer Relations will return $-3+4x^2$ as the smallest polynomial with the initial input as a root. By solving the equation $-3+4x^2=0$, the students will discover that the ratio of the altitude to one of the sides is indeed $\sqrt{3}/2$.

1.3 Conclusion

A natural follow-up to this activity would be a discussion and investigation of what effect the input precision should have on the evaluation precision. Through trial and error, students will find that inappropriate precision specifications can give rise to misleading results.

Cassiopeia (left) made by CASIO and MRI Graphing Calculator Windows CE edition (right) developed by MathResources are a powerful mobile tool set for mathematics students, educators and researchers.

www.casio.com/education/

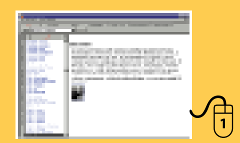
An Investigation of One of Euler's Famous Infinite Sums

2.0 Introduction

Leonhard Euler (1707-1783) was the first person to show that the function $Zeta(2)$, $\sum_{1}^{\infty}\frac{1}{n^2}$, is equal to $\frac{\pi^2}{6}$, in a proof that had eluded other prominent mathematicians including the Bernoulli brothers. An activity involving Euler's infinite sum can be used at several levels: at the secondary level as motivation for introducing sigma notation and the concept of an infinite sum; or at the undergraduate level as part of a course in linear algebra.

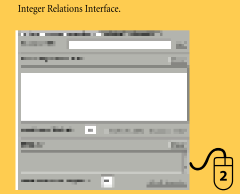
2.1 Using the MathResource (*www.mathresources.com*)

Teachers can introduce the history of the infinite sum problem by having the students conduct a web search or consult a relevant database such as *The MathResource*.



2.2 Using Integer Relations

Students who are seeking a linear dependence relation may enter data into the fields in the *Linear Integer Dependence* panel of the Integer Relations Interface.



2.2.1 Testing a Hypothesis with Integer Relations

In this case, the students are testing the hypothesis that $Zeta(2)$ $\sum_{1}^{\infty}\frac{1}{n^2}$ is related to a power of π.

2.2.2 Applying Results from Linear Integer Dependence

In this case, Integer Relations returns the vector $[-6,0,1,0]$. This implies that $-6\cdot Zeta(2)+0\cdot\pi+1\cdot\pi^2+0\cdot 1=0$. Solving for $Zeta(2)$, the students are led to the conclusion that $Zeta(2)=\pi^2/6$.

2.3 Conclusion

As with the previous activity, this exercise lends itself to a discussion of how the specification of the input parameters affect the output. Most students can easily be motivated to experiment with the interface. For example, students can be challenged to find another value of s for which $Zeta(s)$ depends on powers of π.

Graph Explorer and JavaView

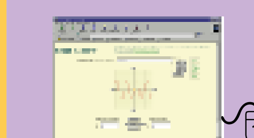
Java-based Math Resources for Students, Educators and Researchers

3.0 Introduction

Graph Explorer and *JavaView* capitalize on Java's cross platform portability to provide sophisticated mathematical tool sets in web-based environments. Graph Explorer is a customizable 2D function plotter while JavaView is a 3D geometry viewer coupled with a comprehensive numerical software library.

3.1 Graph Explorer (*www.langara.bc.ca/mathstats/resource/GraphExplorer*)

Graph Explorer is a resource for math students and educators, and it illustrates how Java can be seriously used over the web in college mathematics. This interface displays function graphs with animated zoom and pan of both graphs and coordinate grid. It also allows users to smoothly vary the values of numerical parameters which it detects in the input formulas. It is being used to experiment with various user interfaces, with the use of the Java Bean component model for rapid prototyping, and with server-based computation in support of applet processes.

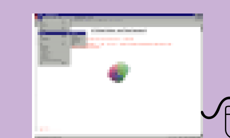


3.2 JavaView (*www-sfb288.math.tu-berlin.de/vgp/javaview/*)

Developed by our partners in Berlin, JavaView is a sophisticated 3D package that allows true visualization on the web. It is sufficiently flexible as it can "grab" *Maple* and other packages, enhance their output and return that enhanced output to the originating software.

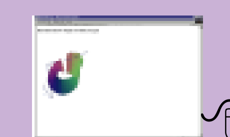
3.2.1 Creating Plots in Maple for Export

The next diagram is a standard Maple plot of a Möbius strip. One of the features of Maple plots is that users can view the plots dynamically by rotating the plot and/or adjusting the plot parameters. Unfortunately, these dynamic attributes are lost when the users save the Maple worksheet as an *html* file. That page merely contains static *gif* images of the state of the users' plot upon export. In other words, the users would not be able to view Maple plots dynamically without Maple. That has changed with the introduction of JavaView.



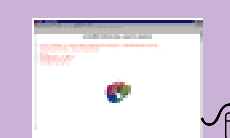
3.2.2 Importing Plots into JavaView

In JavaView, the viewing capabilities are far superior to those of Maple. Not only can the users rotate the geometry, they can also scale, translate, animate or pull the geometry apart. In fact, *JavaViewLib* exports the users' Maple plots to a *dynamic html* page so that the dynamicism created in Maple is preserved upon export for use on the web.



3.2.3 Making Modification s with JavaView

By using JavaViewLib, the users can easily export a plot of the Möbius strip, modify it and import it back into Maple for standard viewing.



3.3 Conclusion

Both Graph Explorer and JavaView function as visualization tools: Graph Explorer in the 2D realm and JavaView in the 3D realm. While computer-based visualization has received attention at the research level for a long time, mathematics educators are now beginning to recognize how the proper use of these tools can benefit their students.

conclusion

Our experience suggests that satisfactory all-purpose proprietary tools are unlikely to develop for most math or science courses. Educators at all levels demand the ability to adapt the resources at their disposal to best meet the needs of their students and to best reflect the way they want to present their curricula. The most desirable *resources* need to be robust, independent, reusable, and ideally, capable of communicating with other resources. Full on-line packaged courses will remain a snare and delusion, but good components are starting to become available. The challenge is to integrate tools and to educate software developers so that this is made as easy as possible.

Waterloo Maple
ADVANCING MATHEMATICS
www.maplesoft.com

Springer
www.cinderella.de

teleport.techbc.ca

www.mathresources.com

www.cecm.sfu.ca